Dutch Disease, Unemployment and Structural Change

Online Appendix

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November 2025

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A The Model

Our framework extends the canonical open economy model of tradable and non-tradable sectors (Schmitt-Grohé and Uribe, 2017, Ch. 8) introducing a commodity sector as in Kulish and Rees (2017), and embedding search and matching frictions in the labor market as in Diamond (1982), Mortensen (1982), and Pissarides (1985).

A.1 Households

The preferences of a typical household in the small open economy are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left\{ \ln\left(C_t - hC_{t-1}\right) - \frac{\tilde{L}_t^{1+\nu}}{1+\nu} \right\}$$

where $\mathbb{E}_0$ denotes the time 0 conditional expectation, β is the household's discount factor, C_t is consumption, $h \in [0,1]$ governs the degree of external habit formation, and ν is the inverse Frisch elasticity of labor supply. The variable ζ_t is an intertemporal preference shock that follows the stochastic process:

$$\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \varepsilon_{\zeta,t} \tag{1}$$

with $\varepsilon_{\zeta,t}$ independently and identically distributed $N(0, \sigma_\zeta^2)$.

Labor supply is a Constant Elasticity of Substitution (CES) aggregate of the household members employed in the tradable sector, $L_{H,t}$, the non-tradable sector, $L_{N,t}$, and the commodity-exporting sector, $L_{X,t}$:

$$\tilde{L}_t = \left(\xi_{H,t} L_{H,t}^{1+\omega} + \xi_{N,t} L_{N,t}^{1+\omega} + \xi_X L_{X,t}^{1+\omega} \right)^{\frac{1}{1+\omega}} \tag{2}$$

Workers view employment in different sectors as imperfect substitutes and the parameter ω reflects the willingness of workers to move between sectors in response to wage differentials.

Households enter the period with $K_{j,t}$ units of capital from sector $j \in \{H, N, X\}$ and B_t^* units of one-period risk-free bonds denominated in foreign currency. During the period, the household receives wages, returns on capital and profits. The household uses its income to purchase new bonds, to invest in new capital and to purchase consumption goods. The resulting flow budget constraint is:

$$C_t + P_{I,t} I_t + P_t^* B_t^* \leq \left(1 + R_{t-1}\right) P_t^* B_{t-1}^* + \sum_{j \in \{H,N,X\}} \left[W_{j,t} L_{j,t} + R_{j,t}^K K_{j,t} \right]$$

where $P_{I,t}$ is the relative price of the investment good in terms of final consumption good, I_t is investment, $W_{j,t}$ is the wage rate in sector j, $R^K_{j,t}$ is the rate of return on capital in sector j, R_t is the interest rates on risk-free bonds and P^*_t is the real exchange rate.

The capital stock of each sector evolves according to the law of motion:

$$K_{j,t+1} = (1-\delta)\,K_{j,t} + V_t\left[1 - \Upsilon\left(\frac{\mathcal{I}_{j,t}}{\mathcal{I}_{j,t-1}}\right)\right]\mathcal{I}_{j,t} \tag{3}$$

for $j \in \{H, N, X\}$, where δ is the common capital depreciation rate and Υ is an investment adjustment cost with the standard restrictions that in steady state $\Upsilon(\bullet) = \Upsilon'(\bullet) = 0$ and $\Upsilon''(\bullet) > 0$. V_t governs the efficiency with which investment adds to the capital stock. It follows the process:

$$V_t = v\left(\frac{1}{z_I}\right)^t \tilde{V}_t$$

where z_I is the differential between the growth rate of real investment and the growth rate of labor-augmenting technology, z. $\tilde{V}_t$ is a stationary autoregressive process that affects the marginal efficiency of investment of the form:

$$\log \tilde{V}_t = \rho_V \log \tilde{V}_{t-1} + \varepsilon_{V,t} \tag{4}$$

where $\varepsilon_{V,t}$ is identically and independently distributed $N(0, \sigma^2_V)$.

The interest rate on risk-free foreign bonds evolves according to the following relation:

$$(1+R_t) = (1+R^*_t)\exp\left[-\psi_b\left(\frac{S_t B^*_t}{Y_t} - b^*\right) + \tilde{\psi}_{b,t}\right] \tag{5}$$

where R^*_t is the foreign interest rate, Y_t is the aggregate output level, and b^* is the steady-state net foreign asset-to-output ratio. $\tilde{\psi}_{b,t}$ is a risk-premium shock which follows the stationary autoregressive process:

$$\tilde{\psi}_{b,t} = (1-\rho_\psi)\tilde{\psi}_b + \rho_\psi \tilde{\psi}_{b,t-1} + \varepsilon_{\psi,t} \tag{6}$$

with $\varepsilon_{\psi,t}$ independently and identically distributed $N(0, \sigma^2_\psi)$.

Consumption Preferences The final consumption good, C_t, is a CES bundle of non-tradable and tradable consumption goods given by

$$C_t = \left[\gamma_{T,t}^{\frac{1}{\eta}} C_{T,t}^{\frac{\eta-1}{\eta}} + \gamma_{N,t}^{\frac{1}{\eta}} C_{N,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where $C_{N,t}$ is the output of the non-tradable sector that is directed towards consumption and has relative price $P_{N,t}$ while $C_{T,t}$ is the output of the tradable sector that is directed towards consumption and has relative price $P_{T,t}$.

$C_{T,t}$ is a composite of domestically-produced and imported tradable goods assembled according to the technology:

$$C_{T,t} = \frac{(C_{H,t})^{\gamma_H}(C_{F,t})^{\gamma_F}}{(\gamma_H)^{\gamma_H}(\gamma_F)^{\gamma_F}}$$

The Cobb-Douglas specification guarantees that the expenditure shares in the tradable consumption basket remain constant.

The non-tradable, domestically-produced tradable and imported consumption goods are all bundles of a continuum of imperfectly substitutable goods:[1]

$$C_{j,t} \equiv \left(\int_0^1 C_{j,t}(i)^{\frac{\theta_j - 1}{\theta_j}} di \right)^{\frac{\theta}{\theta - 1}}$$

for $j \in \{H, N, F\}$. Profit maximisation and the zero-profit condition imply that mplies that the non-tradable consumption good's relative price and the tradable consumption good's relative price evolve according to:

$$1 = \left[\gamma_{T,t} P_{T,t}^{1-\eta} + \gamma_{N,t} P_{N,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{7}$$

and the relative price of the tradable consumption good is a Cobb-Douglas aggregate of the relative prices of home-produced and imported goods:

$$P_{T,t} = (P_{H,t})^{\gamma_H} (P_{F,t})^{\gamma_F} \tag{8}$$

Investment Preferences The final investment good, I_t, is a Cobb-Douglas bundle of non-tradable and tradable investment goods given by

$$I_t = z_v^t \frac{(I_{T,t})^{\gamma_T^I}(I_{N,t})^{\gamma_N^I}}{(\gamma_T^I)^{\gamma_T^I}(\gamma_N^I)^{\gamma_N^I}}$$

where $I_{N,t}$ is the output of the non-tradable sector directed towards the production of investment, $I_{T,t}$ is the output of the tradable sector that is directed towards investment and z_v is a productivity trend that jointly with the growth rates of $I_{T,t}$ and $I_{N,t}$ determines

[1] This is also the case for investment, $I_{j,t}$ for $j \in \{H, N, F\}$.

the steady state growth rate of final investment, z_I. $I_{T,t}$ is a composite of domestically-and foreign produced tradable goods that is assembled according to the technology:

$$I_{T,t} = \frac{(I_{H,t})^{\gamma_H^I}(I_{F,t})^{\gamma_F^I}}{(\gamma_H^I)^{\gamma_H^I}(\gamma_F^I)^{\gamma_F^I}}$$

The corresponding price indices are:

$$P_t^I = z_v^{-t}(P_{T,t}^I)^{\gamma_T^I}(P_{N,t})^{\gamma_N^I} \tag{9}$$

and

$$P_{T,t}^I = (P_{H,t})^{\gamma_H^I}(P_{F,t})^{\gamma_F^I} \tag{10}$$

As the shares of non-tradable, domestically-produced tradable and imported goods in the investment and consumption composites differ, the relative price of the investment good, $P_{I,t}$, will, in general, differ from 1. Similarly, the relative price of tradable consumption goods, $P_{T,t}$, will differ from the relative price of tradable investment goods, $P_{T,t}^I$.

A.2 Search and Matching in the Labor Market

Assuming full participation in the labor market, then the pool of unemployed household members, U_t, is given as:

$$U_t = 1 - L_t \tag{11}$$

where $L_t = L_{H,t} + L_{N,t} + L_{X,t}$. Of the pool of unemployed household members, $U_{H,t}$, $U_{N,t}$ and $U_{X,t}$ household members search for a job in the tradable, non-tradable and commodities sectors, respectively:

$$U_t = U_{H,t} + U_{N,t} + U_{X,t} \tag{12}$$

The presence of search and matching frictions in the labor market prevents some unemployed household members from finding jobs. Employment in each production sector $j \in \{H, N, X\}$ evolves according to:

$$L_{j,t} = (1 - \Phi_j)L_{j,t-1} + H_{j,t-1} \tag{13}$$

where $\Phi_j \in [0, 1]$ is an exogenous separation rate in production sector j and $H_{j,t-1}$ represents the measure of workers hired by production sector j in period $t - 1$.

The jobs that get destroyed in sector j at time t add to unemployment in that sector, but then switching happens from the pool of the unemployed in that sector. For example, the unemployed at time t in sector H, $U_{H,t}$, includes a fraction that were already unemployed in the sector and remain there, $\pi_{H,H}U_{H,t-1}$, a fraction that switched from the other sectors, $\pi_{N,H}U_{N,t-1}$ and $\pi_{X,H}U_{X,t-1}$, those whose jobs get destroyed in just that sector, $\Phi_H L_{H,t-1}$, less those who find a job in the sector, $H_{H,t-1}$. So, we have

$$U_{H,t} = \pi_{HH}U_{H,t-1} + \pi_{NH}U_{N,t-1} + \pi_{XH}U_{X,t-1} + \Phi_H L_{H,t-1} - H_{H,t-1} \quad (14)$$

$$U_{N,t} = \pi_{HN}U_{H,t-1} + \pi_{NN}U_{N,t-1} + \pi_{XN}U_{X,t-1} + \Phi_N L_{N,t-1} - H_{N,t-1} \quad (15)$$

$$U_{X,t} = \pi_{HX}U_{H,t-1} + \pi_{NX}U_{N,t-1} + \pi_{XX}U_{X,t-1} + \Phi_X L_{X,t-1} - H_{X,t-1} \quad (16)$$

where for each sector $j \in \{H, N, X\}$, we have that the transition probabilities sastify $\sum_{k\in\{H,N,X\}} \pi_{j,k} = 1$.

New matches in the labor market are determined by Cobb-Douglas matching functions:

$$H_{j,t} = \chi_j \zeta_t^{\chi} U_{j,t}^{\mu_j} V_{j,t}^{1-\mu_j} \quad (17)$$

where $U_{j,t}$ denotes the number of unemployed household members searching for a job in production sector j, $V_{j,t}$ denotes the number of vacancies available in production sector j, μ_j is the matching elasticity with respect to unemployment, and χ_j is the matching efficiency in sector j. ζ_t^{χ} is a matching efficiency shock common to all sectors which follows the stationary autoregressive process:

$$\zeta_t^{\chi} = \rho_{\chi}\zeta_{t-1}^{\chi} + \varepsilon_{\chi,t} \quad (18)$$

with $\varepsilon_{\chi,t}$ independently and identically distributed $N(0, \sigma_{\chi}^2)$.

From the matching functions, we define the vacancy filling rate in sector j, $M_{j,t}$, as:

$$M_{j,t} = \frac{H_{j,t}}{V_{j,t}} \quad (19)$$

and the job finding rate conditional on searching in a particular production sector j, $S_{j,t}$, as:

$$S_{j,t} = \frac{H_{j,t}}{U_{j,t}} \quad (20)$$

A.3 Intermediate Goods Producing Firms

The economy features four intermediate good producers: commodity firms, non-tradable firms, domestic tradable firms and importing firms. We describe each in turn.

A.3.1 Commodity-Exporting Firms

Commodity firms produce a homogeneous good using the Cobb-Douglas production function:

$$Y_{X,t} = Z_{X,t} K_{X,t}^{\alpha_X} \left(Z_t L_{X,t} \right)^{1-\alpha_X} \tag{21}$$

where Z_t is a labor-augmenting technology shock, common to all producing sectors. Its growth rate, $z_t = Z_t / Z_{t-1}$, follows the process:

$$\log z_t = (1-\rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_{z,t} \tag{22}$$

where $z > 1$ determines the trend growth rate of real GDP and $\varepsilon_{z,t}$ is independently and identically distributed $N(0, \sigma_z^2)$. The sector-specific productivity process, $Z_{X,t}$, follows:

$$Z_{X,t} = z_X^t \tilde{Z}_{X,t}$$

where z_X determines the differential growth rate, along the balanced growth path, between the output of the commodity-exporting sector and real GDP and $\tilde{Z}_{X,t}$ follows the process:

$$\log \tilde{Z}_{X,t} = \rho_X \log \tilde{Z}_{X,t-1} + \varepsilon_{X,t} \tag{23}$$

where $\varepsilon_{X,t}$ is independently and identically distributed $N(0, \sigma_X^2)$.

Commodity producing firms face a cost to posting vacancies as well as a cost for adjusting the number of posted vacancies of the form:

$$\Psi_{V,X}(V_{X,t}, V_{X,t-1}) = \psi_{VX,t} V_{X,t} + \frac{\psi'_{VX,t}}{2} \left(\frac{V_{X,t}}{V_{X,t-1}} - 1 \right)^2 V_{X,t}$$

where $V_{X,t}$ is the number of vacancies posted in the commodities sector.

The real exchange rate is defined as the relative price of the foreign consumption bundle, P_t^*, in terms of the domestic consumption bundle, whose price we normalise to unity. Firms in the commodity sector export commodities at a price set by the world market and the relative price of commodities is assumed to follow:

$$P_{X,t} = \kappa_t P_t^*, \tag{24}$$

where κ_t governs the relative price of commodities that is determined by

$$\log \kappa_t = (1-\rho_\kappa) \log \kappa + \rho_\kappa \log \kappa_{t-1} + \varepsilon_{\kappa,t}, \tag{25}$$

where $\varepsilon_{\kappa,t} \sim N(0, \sigma_\kappa^2)$ is a white noise shock with variance σ_κ^2, and the parameter κ governs the long-run level of commodity prices that is one of the determinants the terms of trade and the steady state of the economy. As in Kulish and Rees (2017), we allow for a break in the long-run level of commodity prices. At an estimated date, the long-run level of commodity prices increases in an unanticipated way and permanently to $\kappa' = \kappa + \Delta_\kappa$. To guard against the possibility that the exogenous increase in commodity prices Δ_κ is instead picking up an increase in volatility, we allow for a break in volatility and assume that the volatility of shocks to commodity prices may change from σ_κ to σ'_κ, at an estimated date. Importantly, in estimation, these changes are allowed but not imposed.

A.3.2 Non-tradable Goods Producing Firms

Non-tradable firms sell differentiated products, which they produce using the Cobb-Douglas production function:

$$Y_{N,t} = Z_{N,t} K_{N,t}^{\alpha_N} \left(Z_t L_{N,t}\right)^{1-\alpha_N} \tag{26}$$

$Z_{N,t}$ is sector-specific productivity process that follows:

$$Z_{N,t} = z_N^t \tilde{Z}_{N,t}$$

where z_N determines the differential growth rate, along the balanced growth path, between the output of the non-tradable sector and real GDP and $\tilde{Z}_{N,t}$ follows the process:

$$\log \tilde{Z}_{N,t} = \rho_N \log \tilde{Z}_{N,t-1} + \varepsilon_{N,t} \tag{27}$$

where $\varepsilon_{N,t}$ is independently and identically distributed $N(0, \sigma_N^2)$.

Firms in the non-tradable sector face a cost to posting vacancies as well as a cost for adjusting the number of posted vacancies of the form:

$$\Psi_{V,N}(V_{N,t}, V_{N,t-1}) = \psi_{VN,t} V_{N,t} + \frac{\psi'_{VN,t}}{2} \left(\frac{V_{N,t}}{V_{N,t-1}} - 1\right)^2 V_{N,t}$$

where $V_{N,t}$ is the number of vacancies posted in the non-tradable sector.

A.3.3 Domestic Tradable Goods Producing Firms

Domestic tradable firms produce differentiated products using the Cobb-Douglas production function:

$$Y_{H,t} = Z_{H,t} K_{H,t}^{\alpha_H} \left(Z_t L_{H,t}\right)^{1-\alpha_H} \tag{28}$$

$Z_{H,t}$ is a stationary sector-specific productivity process that follows:

$$Z_{H,t} = z_H^t \tilde{Z}_{H,t}$$

where $z_H > 0$ determines the differential growth rate, along the balanced growth path, between the output of the tradable sector and real GDP and $\tilde{Z}_{H,t}$ follows the process:

$$\log \tilde{Z}_{H,t} = \rho_H \log \tilde{Z}_{H,t-1} + \varepsilon_{H,t} \tag{29}$$

where $\varepsilon_{H,t}$ is independently and identically distributed $N(0, \sigma_H^2)$.

Like their non-tradable counterparts, tradable firms also face a cost to posting vacancies as well as a cost for adjusting the number of posted vacancies of the form:

$$\Psi_{V,H}(V_{H,t}, V_{H,t-1}) = \psi_{VH,t} V_{H,t} + \frac{\psi'_{VH,t}}{2} \left(\frac{V_{H,t}}{V_{H,t-1}} - 1 \right)^2 V_{H,t}$$

where $V_{H,t}$ is the number of vacancies posted in the tradable sector.

A.3.4 Importing Firms

Importing firms act as retailers by purchasing foreign-manufactured goods at the relative price P_t^* and reselling them in the domestic market at relative price $P_{F,t}$.[2] The importing firm's optimisation problem yields

$$P_{F,t} = P_t^* \tag{30}$$

A.4 Wage Determination

The value for a household member of being employed in production sector $j \in \{H, N, X\}$ is given by:

$$\mathcal{V}_{j,t} = W_{j,t} - \frac{\zeta_t \xi_{j,t} L_{j,t}^{\omega} \tilde{L}_t^{\nu-\omega}}{\Lambda_t} + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-\Phi_j)\mathcal{V}_{j,t+1} + \Phi_j \mathcal{U}_{j,t+1} \right] \right\} \tag{31}$$

where Λ_t is the stochastic discount factor and $\mathcal{U}_{j,t}$ is the value of being unemployed in production sector j.

The value for a household member of being unemployed in production sector $j \in \{H, N, X\}$ is given by:

$$\mathcal{U}_{j,t} = \beta E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left\{ S_{j,t} \mathcal{V}_{j,t+1} + (1 - S_{j,t}) \left[\pi_{jj} \mathcal{U}_{j,t+1} + \sum_{i \neq j} \pi_{ji} \mathcal{U}_{i,t+1} \right] \right\} \right) \tag{32}$$

[2] We assume that the price of the consumption good in the rest of the world relative to the price of imports is constant and set it to unity (i.e., $P_t^* = P_{F,t}^*$)

The value of a job to firm in production sector $j \in \{H, N, X\}$ is given by:

$$\mathcal{J}_{j,t} = \left((1-\alpha_j) \frac{P_{j,t} Y_{j,t}}{L_{j,t}} - W_{j,t} \right) + \beta(1-\Phi_j) E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \mathcal{J}_{j,t+1} \right\} \tag{33}$$

Finally, we assume that wages in the three production sectors are set through Nash bargaining. The Nash bargaining solution yields the wage rate that maximizes the weighted product of the worker's and firm's net return from the job match in each sector. The first-order condition from this maximization problem is:

$$\Omega_j \mathcal{J}_{j,t} = (1-\Omega_j)(\mathcal{V}_{j,t} - \mathcal{U}_{j,t}) \tag{34}$$

where the parameter Ω_j represents the worker's bargaining power in sector j.

A.5 Foreign Sector, Net Exports and the Current Account

The foreign demand function for domestically produced tradable goods, $C^*_{H,t}$, is of the form:

$$C^*_{H,t} = \gamma^*_{H,t} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\eta^*} \tilde{Y}^*_t \tag{35}$$

Foreign output, $\tilde{Y}^*_t$, follows the non-stationary process:

$$\tilde{Y}^*_t = Z_t (z^*)^t Y^*_t$$

where z^* is the differential growth rate of foreign output. The foreign interest rate, R^*_t, is assumed to follow the process:

$$\ln(1+R^*_t) = (1-\rho_{R^*}) \ln(1+R^*) + \rho_{R^*} \ln(1+R^*_{t-1}) + \varepsilon_{R^*,t} \tag{36}$$

where $\varepsilon_{R^*,t}$ is independently and identically distributed $N(0, \sigma^2_{R^*})$.

Net exports are given by:

$$NX_t = P_{H,t} C^*_{H,t} + P_{X,t} Y_{X,t} - P_{F,t} Y_{F,t} - P_{X,t} \Psi_{V,X}(V_{X,t}, V_{X,t-1}) \tag{37}$$

and so, the current account equation is given by:

$$S_t \left(B^*_t - B^*_{t-1} \right) = R_{t-1} S_t B^*_{t-1} + NX_t \tag{38}$$

A.6 Market Clearing

For investment goods, market clearing implies that the quantity produced of these goods equals the demand for them by the production sectors:

$$I_t = \mathcal{I}_{H,t} + \mathcal{I}_{N,t} + \mathcal{I}_{X,t} \tag{39}$$

Market clearing also requires that the quantity of goods produced in the non-tradable sector, the tradable sector, and the imports sector is equal to the quantity demanded for these goods:

$$Y_{N,t} = C_{N,t} + I_{N,t} + \Psi_{V,N}(V_{N,t}, V_{N,t-1}) \tag{40}$$

$$Y_{H,t} = C_{H,t} + C^*_{H,t} + I_{H,t} + \Psi_{V,H}(V_{H,t}, V_{H,t-1}) \tag{41}$$

$$Y_{F,t} = C_{F,t} + I_{F,t} \tag{42}$$

Finally, aggregate output is defined as:

$$Y_t = P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} + P_{X,t}Y_{X,t} \tag{43}$$

B Derivation of the Wage Equation

The Nash bargaining solution in sector j yields the following first-order condition:

$$\Omega_j \mathcal{J}_{j,t} = (1-\Omega_j)(\mathcal{V}_{j,t} - \mathcal{U}_{j,t}) \tag{44}$$

where

$$\mathcal{V}_{j,t} = W_{j,t} - \frac{\zeta_t \xi_{j,t} L_{j,t}^{\omega} \tilde{L}_t^{\nu-\omega}}{\Lambda_t} + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-\Phi_j)\mathcal{V}_{j,t+1} + \Phi_j \mathcal{U}_{j,t+1} \right] \right\} \tag{45}$$

$$\mathcal{U}_{j,t} = \beta E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left\{ S_{j,t}\mathcal{V}_{j,t+1} + (1-S_{j,t}) \left[\pi_{jj}\mathcal{U}_{j,t+1} + \sum_{i\neq j} \pi_{ji}\mathcal{U}_{i,t+1} \right] \right\} \right) \tag{46}$$

$$\mathcal{J}_{j,t} = \left((1-\alpha_j)\frac{P_{j,t}Y_{j,t}}{L_{j,t}} - W_{j,t} \right) + \beta(1-\Phi_j) E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \mathcal{J}_{j,t+1} \right\} \tag{47}$$

Subtracting equation (46) from equation (45) gives:

$$\begin{aligned}\mathcal{V}_{j,t} - \mathcal{U}_{j,t} = W_{j,t} - \frac{\zeta_t \xi_{j,t} L_{j,t}^{\omega} \tilde{L}_t^{\nu-\omega}}{\Lambda_t} + \beta(1-\Phi_j - S_{j,t}) E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\mathcal{V}_{j,t+1} - \mathcal{U}_{j,t+1} \right] \right\} \\ + \beta(1-S_{j,t}) E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\sum_{i\neq j} \pi_{ji} \left(U_{j,t+1} - U_{i,t+1} \right) \right] \right\}\end{aligned} \tag{48}$$

The Nash bargaining condition holds for every period, so we can write:

$$\mathcal{V}_{j,t+1} - \mathcal{U}_{j,t+1} = \frac{\Omega_j}{1-\Omega_j} \mathcal{J}_{j,t+1}$$

which when substituted into equation (48) yields:

$$\begin{aligned}\mathcal{V}_{j,t} - \mathcal{U}_{j,t} = W_{j,t} - \frac{\zeta_t \xi_{j,t} L_{j,t}^{\omega} \tilde{L}_t^{\nu-\omega}}{\Lambda_t} + \beta(1-\Phi_j - S_{j,t}) \frac{\Omega_j}{1-\Omega_j} E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \mathcal{J}_{j,t+1} \right\} \\ + \beta(1-S_{j,t}) E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\sum_{i\neq j} \pi_{ji} \left(U_{j,t+1} - U_{i,t+1} \right) \right] \right\}\end{aligned} \tag{49}$$

The vacancy posting condition derived from the firm's optimisation problem is given by:

$$\beta M_{j,t} E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \tilde{\mathcal{J}}_{j,t+1} \right\} = \frac{\partial \Psi_{V,j}(V_{j,t}, V_{j,t-1})}{\partial V_{j,t}} + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial \Psi_{V,j}(V_{j,t+1}, V_{j,t})}{\partial V_{j,t}} \right\} \tag{50}$$

Substituting equation (50) into equations (47) and (49) gives:

$$\mathcal{J}_{j,t} = (1-\alpha_j)\frac{P_{j,t}Y_{j,t}}{L_{j,t}} - W_{j,t} + \frac{1-\Phi_j}{M_{j,t}} \left(\frac{\partial \Psi_{V,j}(V_{j,t}, V_{j,t-1})}{\partial V_{j,t}} + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial \Psi_{V,j}(V_{j,t+1}, V_{j,t})}{\partial V_{j,t}} \right\} \right) \tag{51}$$

and

$$\mathcal{V}_{j,t} - \mathcal{U}_{j,t} = W_{j,t} - \frac{\zeta_t \xi_{j,t} L_{j,t}^{\omega} \tilde{L}_t^{\nu-\omega}}{\Lambda_t} + \frac{1 - \Phi_j - S_{j,t}}{M_{j,t}} \frac{\Omega_j}{1 - \Omega_j} \Bigg(\frac{\partial \Psi_{V,j}(V_{j,t}, V_{j,t-1})}{\partial V_{j,t}} \tag{52}$$

$$+ \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial \Psi_{V,j}(V_{j,t+1}, V_{j,t})}{\partial V_{j,t}} \right\} \Bigg) + \beta(1 - S_{j,t}) E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\sum_{i \neq j} \pi_{ji} \left(U_{j,t+1} - U_{i,t+1} \right) \right] \right\}$$

Finally, plugging equations (51) and (52) into the Nash bargaining condition given in equation (44) and rearranging gives the wage equation:

$$W_{j,t} = \Omega_j \left\{ (1 - \alpha_j) \frac{P_{j,t} Y_{j,t}}{L_{j,t}} + \theta_{j,t} \left[\frac{\partial \Psi_{V,j}(V_{j,t}, V_{j,t-1})}{\partial V_{j,t}} + \beta E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial \Psi_{V,j}(V_{j,t+1}, V_{j,t})}{\partial V_{j,t}} \right) \right] \right\}$$

$$+ (1 - \Omega_j) \left\{ \frac{\zeta_t \xi_{j,t} L_{j,t}^{\omega} \tilde{L}_t^{\nu-\omega}}{\Lambda_t} - \beta(1 - S_{j,t}) E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\sum_{i \neq j} \pi_{ji} \left(U_{j,t+1} - U_{i,t+1} \right) \right) \right] \right\} \tag{53}$$

where $\theta_{j,t} = S_{j,t} / M_{j,t}$ is the labor market tightness in production sector j.

C Modelling Structural Drifts

C.1 Structural Change in Consumption Preferences

The consumption preference shifters $\gamma_{N,t}$ and $\gamma_{T,t}$ with stochastic (superscript s) and deterministic (superscript d) components, follow the processes:

$$\gamma_{N,t} = \gamma^s_{N,t}\gamma^d_{N,t} \tag{54}$$

$$\gamma_{T,t} = 1 - \gamma_{N,t} \tag{55}$$

where the stochastic component follows a process so as to obtain a balanced growth given the productivity differentials:

$$\gamma^s_{N,t} = Z^{1-\eta}_{N,t} \tag{56}$$

and the deterministic component follows an anticipated sequence $\{\gamma^d_{N,t}\}^{\infty}_{t=0}$ that is known to agents from the start and which we define by:

$$\gamma^d_{N,t} = \gamma^d_{N,t-1} + \Delta\gamma_N \tag{57}$$

where the parameter $\Delta\gamma_N$ determines the speed of the drift in the composition of final consumption.

C.2 Structural Change in Employment Preferences

The labor preferences shifters $\zeta_{H,t}$ and $\zeta_{N,t}$ are comprised of stochastic (superscript s) and deterministic (superscript d) components, which follow the processes:

$$\zeta_{H,t} = \zeta^s_{H,t}\zeta^d_{H,t}, \tag{58}$$

$$\zeta_{N,t} = \zeta^s_{N,t}\zeta^d_{N,t}, \tag{59}$$

where the stochastic components follow standard stationary autoregressive processes:

$$\ln\zeta^s_{H,t} = \rho_H \ln\zeta^s_{H,t-1} + \varepsilon_{\zeta_H,t} \tag{60}$$

$$\ln\zeta^s_{N,t} = \rho_N \ln\zeta^s_{N,t-1} + \varepsilon_{\zeta_N,t} \tag{61}$$

and the deterministic components follow the anticipated sequences $\{\zeta^d_{H,t}\}^{\infty}_{t=0}$ and $\{\zeta^d_{N,t}\}^{\infty}_{t=0}$ that are known to agents from period $t = 0$. The anticipated sequences are defined by:

$$\zeta^d_{H,t} = \zeta^d_{H,t-1} + \Delta_{\zeta_H}, \tag{62}$$

$$\zeta^d_{N,t} = \zeta^d_{N,t-1} + \Delta_{\zeta_N}, \tag{63}$$

where Δ_{ξ_H} and Δ_{ξ_N} are in turn defined by:

$$\Delta_{\xi_H} = \frac{\xi_{H,0}}{T}(\Delta_{\xi} - 1), \tag{64}$$

$$\Delta_{\xi_N} = \frac{\xi_{N,0}}{T}\left(\frac{1}{\Delta_{\xi}} - 1\right), \tag{65}$$

where the parameter Δ_{ξ} determines the speed of the drifts in employment preferences.

C.3 Solution Method

We apply the general method proposed by Kulish and Pagan (2017) to solve models under structural change. Our application incorporates structural changes that are *anticipated* (the changes in preferences over the disutility of work and consumption across goods in different sectors) and *unanticipated* (the changes in the level and volatility of commodity prices).

We assume that the *anticipated* structural changes start and end out-of-sample. The changes in the disutility of working in the tradable and non-tradable sectors, $\xi^d_{H,t}$ and $\xi^d_{N,t}$, and the changes in the preferences over non-tradable goods, $\gamma^d_{N,t}$, are anticipated by agents before the start of our sample, as illustrated in panels (a) and (b) of Figure 1.

We also assume one-off *unanticipated* and permanent changes in the long-run level of commodity prices, κ, at an estimated date, T_κ, and in the volatility of commodity prices σ^2_κ, at an estimated date, T_σ.[3] We restrict the unanticipated changes to take place within the sample period, as illustrated in panel (c) of Figure 1.

Next, we describe the anticipated structural changes, represented by the sequence of parameters determined by the following simplified equations for the purposes of demonstration:[4]

$$\begin{aligned}\gamma^d_{N,t} &= \gamma^d_{N,t-1} + \Delta_{\gamma_N},\\ \xi^d_{H,t} &= \xi^d_{H,t-1} + \Delta_{\xi_H},\\ \xi^d_{N,t} &= \xi^d_{N,t-1} + \Delta_{\xi_N}.\end{aligned}$$

These anticipated structural changes start from the initial values $\gamma^d_{N,0}$, $\xi^d_{H,0}$ and $\xi^d_{N,0}$, and

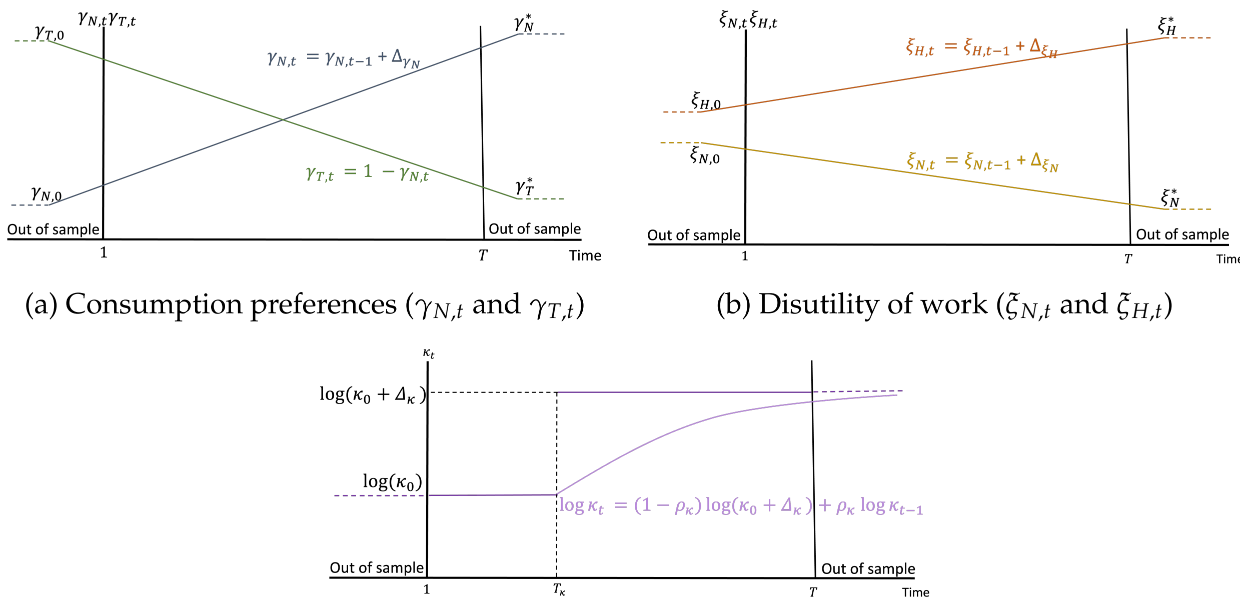
[3]Given the autoregressive process for commodity prices, the break in the long-run level of commodity prices of Δ_κ implies that the non-stochastic path of commodity prices increases gradually over time towards its new long-run value.

[4]See equations (**??**) and (62)-(**??**) for the full specifications of the structural changes.

evolve with drifts Δ_{γ_N}, Δ_{ξ_H} and Δ_{ξ_N}. We estimate the initial values and the drifts that deliver the best match of the data.

Panels (a) and (b) in Figure 1 illustrate how the structural parameters, $\{\gamma^d_{N,t}, \gamma^d_{T,t}\}$ and $\{\xi^d_{T,t}, \xi^d_{N,t}\}$ evolve, given initial conditions $\{\gamma^d_{N,0}\gamma^d_{T,0}\}$, $\{\xi^d_{H,0}, \xi^d_{N,0}\}$ and values for the drift parameters Δ's. Panel (c) in Figure 1 illustrates the process for commodity prices κ_t represented by equation (25). Given the autoregressive process for commodity prices, a break in the long-run level of commodity prices of Δ_κ implies that the non-stochastic transition path of commodity prices increases gradually over time towards its new long-run value. Figure 1 highlights that one could allow the process of structural change to start before the beginning of the sample and to stop after the end of the sample, which is what we do in our estimation.

Figure 1: Structural Changes



(a) Consumption preferences ($\gamma_{N,t}$ and $\gamma_{T,t}$)

(b) Disutility of work ($\xi_{N,t}$ and $\xi_{H,t}$)

(c) Level of commodity prices (κ_t)

Note: The sequences in panels (a) and (b) are anticipated, but the change in the long-run mean of κ to $\kappa_0 + \Delta_\kappa$ in panel (c) is unanticipated.

We assume that the process of structural change ends in period T^*. For each period $t \geq T^*$ the model is described by the non-linear system of equations of stochastically detrended variables Y_t:

$$\mathbb{E}_t F(Y_{t-1}, Y_t, Y_{t+1}, \varepsilon_t, \theta^*, \theta) = 0 \qquad \text{for} \quad t \geq T^*, \tag{66}$$

where $\theta^* = (\xi_H^*, \xi_N^*, \gamma_N^*, \kappa^*)$ are the terminal values of the structural parameters that change, and θ contains the parameters unrelated with structural change. $\mathbb{E}_t$ is the expectation operator and ε_t contains the business cycle shocks. In the absence of shocks, the system (66) has a steady state, Y^*, satisfying $F(Y^*, Y^*, Y^*, 0, \theta^*, \theta) = 0$. Linearising the system (66) around Y^* yields the linear system of equations:

$$A_0^* y_t = C_0^* + A_1^* y_{t-1} + B_0^* \mathbb{E}_t y_{t+1} + D_0^* \varepsilon_t, \tag{67}$$

where $y_t = \ln Y_t$ and the matrices of structural parameters, $A_0^*, A_1^*, B_0^*, C_0^*$ and D_0^* represent the coefficients of the linearization of the terminal *time-invariant* structure. The linear, rational expectations solution to (67) is given by the VAR representation:[5]

$$y_t = C^* + Q^* y_{t-1} + G^* \varepsilon_t. \tag{68}$$

While structural change is undergoing, that is for $t = 1, 2, ..., T^* - 1$, the non-linear system of equations is equal to:

$$\mathbb{E}_t F(Y_{t-1}, Y_t, Y_{t+1}, \varepsilon_t, \theta_t^d, \theta) = 0 \qquad 1 \leq t < T^* \tag{69}$$

where $\theta_t^d = \left(\xi_{H,t}^d, \xi_{N,t}^d, \gamma_{N,t}^d\right)$ is the vector of deterministic *time-varying* preference shifters. For each period $t = 1, 2, ..., T^* - 1$, we solve for the steady state which is implied by the absence of shocks and the assumption that θ_t^d prevails into the indefinite future. Thus, we solve for Y in the system:

$$F(Y, Y, Y, 0, \theta_t^d, \theta) = 0.$$

This steady state is a function of the parameter values that prevail at t, that is θ_t^d, so one can write $Y(\theta_t^d)$. During the period of structural changes, when $t = 1, 2, ..., T^* - 1$, we linearize the model around $Y(\theta_t^d)$ which gives the linearised system:

$$A_{0,t} y_t = C_{0,t} + A_{1,t} y_{t-1} + B_{0,t} \mathbb{E}_t y_{t+1} + D_{0,t} \varepsilon_t, \qquad 1 \leq t < T^*, \tag{70}$$

where the matrices of structural parameters, $A_{0,t}, A_{1,t}, B_{0,t}, C_{0,t}$ and $D_{0,t}$ are time-varying reflecting the fact that the coefficients of the linearization change with the expansion point, $Y(\theta_t^d)$.

Using equations (67) and (70), we solve the model using the following recursive approach. Since the sequence of structural change $\{\theta_t^d\}$ is anticipated, the solution for y_t is a

[5] The condition of existence and uniqueness of the solutions are the same as in Binder and Pesaran (1997).

time-varying VAR of the form:

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t. \tag{71}$$

As agents have perfect foresight of the forthcoming structural changes, the expectation of y_{t+1} is equal to $\mathbb{E}_t y_{t+1} = C_{t+1} + Q_{t+1} y_t$. Using this conditional expectation, we apply the method of undetermined coefficients to obtain:

$$(I - B_t Q_{t+1})^{-1}(\Gamma_t + B_t C_{t+1}) = C_t, \tag{72}$$

$$(I - B_t Q_{t+1})^{-1} A_t = Q_t, \tag{73}$$

$$(I - B_t Q_{t+1})^{-1} D_t = G_t, \tag{74}$$

where $\Gamma_t \equiv A_{0,t}^{-1} C_{0,t}$, $A_t \equiv A_{0,t}^{-1} A_{1,t}$, $B_t \equiv A_{0,t}^{-1} B_{0,t}$ and $D_t \equiv A_{0,t}^{-1} D_{0,t}$. To solve for the sequence of reduced-form matrices, we start from the terminal solution after which there are no more structural changes, that is, $y_t = C^* + Q^* y_{t-1} + G^* \varepsilon_t$ for $t \geq T^*$, and use equation (73) to find the sequence of $\{Q_t\}$ for $t < T^*$. Once we obtain the sequence for $\{Q_t\}$, it is straightforward to find the sequences $\{C_t\}$ and $\{G_t\}$ from equations (72) and (74). Using the solution (71) with the matrices $\{C_t, G_t, Q_t\}$, we derive the likelihood function for the set of observable variables, as described in Kulish and Pagan (2017).

The unanticipated change in the level of commodity prices (κ) is handled as follows: at the time of the change (T_κ), we recompute $Y(\theta_t^d)$, for $t = T_\kappa, ..., T^*$ and re-linearize the system around the updated $Y(\theta_t^d)$ which gives a new set of linearised structural equations:

$$A_{0,t} y_t = C_{0,t} + A_{1,t} y_{t-1} + B_{0,t} \mathbb{E}_t y_{t+1} + D_{0,t} \varepsilon_t, \qquad T_\kappa \leq t < T^*. \tag{75}$$

Using the updated sequence of structural matrices, we proceed as before and recompute using backward recursions the sub-sequence of reduced form matrices, $\{C_t, G_t, Q_t\}$ from T_κ onwards. To guard against the possibility that our estimates capture an increase in the volatility of commodity prices as a permanent increase in the long-run level of commodity prices, we allow for a break in the variance of shocks to commodity prices, in σ_κ. Since we are working with a first-order approximation the unanticipated break in variance has no impact on $\{C_t, Q_t, G_t\}$.[6]

[6] The change in variance is captured as a break of the variance covariance matrix of the structural shocks which affects the likelihood but not the solution under structural change. See the Online Appendix for more details.

D Data Sources

This section describes the data used to estimate the model.

Population: Quarterly gross domestic product in chain volume measure (ABS Catalogue 5206.001) divided by quarterly gross domestic product per capita also in chain volume measure (ABS Catalogue 5206.001).

Consumption per capita: Quarterly private consumption in chain volume measure (ABS Catalogue 5206.002) divided by population. The series enters in first difference in estimation with its sample mean adjusted to match that of real output growth.

Investment per Capita: Quarterly gross fixed capital formation in chain volume measure (ABS Catalogue 5206.002) divided by population. The series enters in first difference in the estimation.

Net exports-to-GDP ratio: Net exports-to-GDP is computed as exports-to-GDP less imports-to-GDP. Exports-to-GDP is quarterly exports in current price measure divided by quarterly gross domestic product in current prices. Imports to-GDP is quarterly imports in current prices divided by quarterly gross domestic product in current prices (ABS Catalogue 5206.003). The sample mean of this series is removed prior to the estimation.

Domestic real interest rate: 90-day bank bill rate (RBA Bulletin Table F1). The nominal interest rate is converted to a real rate using the trimmed mean inflation series (RBA Bulletin Table G1). The monthly series is converted into quarterly frequency by arithmetic averaging.

Real exchange rate: Australian Real Trade-Weighted Index (RBA Bulletin Table F15). The series enters in first difference in the estimation.

Unemployment rate: Monthly Australian unemployment rate (ABS Catalogue 6202.001). The monthly series are converted into quarterly frequency by arithmetic averaging.

Non-tradable consumption share: Non-tradable consumption share is computed as the ratio of nominal non-tradable consumption to aggregate nominal consumption. Non-tradable consumption includes the consumption categories: Rent, Electricity, Gas & Water, Operation of Vehicles, Transport Services, Education, Hotels, Cafes & Restaurants, Insurance & Financial Services as well as Healthcare and Other Households Services (ABS Catalogue 5206.008). The series enters in first difference in the estimation.

Non-tradable employment share: Non-tradable employment share is computed as the ratio of non-tradable employment to aggregate employment. Non-tradable employment is defined as the sum of Utilities, Construction, Retail Trade, Media & Telecommunications, Hiring & Real Estate Services, Financial & Insurance Services, Scientific & Technical Services, Administrative Services, Educational, Health care & Social Assistance, and Arts & Recreation employment. (ABS Catalogue 6291.004).

Commodity prices: Quarterly Commodity Price Index (RBA Bulletin Table I2).

Foreign real interest rate: Foreign interest rate is computed as the average policy rate in the Euro area, the United States, and Japan (RBA Bulletin Table F13). The monthly series are converted into quarterly frequency by arithmetic averaging. German interest rate is used before the introduction of the Euro (FRED Database series INTDSRDEM193N).

E Calibration Results

E.1 First Moments

Target	Average 1985-2019	Average 1985-2002	Model
Macro Aggregates (annual per cent)			
Per capita output growth	1.70	2.18	1.69
Per capita investment growth	2.68	2.25	3.34
Domestic real interest rate	3.07	4.69	3.50
Foreign real interest rate	-0.004	0.78	0.73
Expenditure (per cent of GDP)			
Consumption	74.8	75.3	73.8
Investment	26.0	25.6	25.7
Exports	19.0	17.7	18.3
Investment Basket (per cent of investment)			
Non-tradable investment	67.5	65.8	65.3
Home tradable investment	2.4	9.3	9.4
Imported investment	30.1	24.9	25.3
Exports (per cent of exports)			
Resource exports	42.5	36.6	37.5
Other exports	57.3	63.4	62.5

E.2 Initial Values

Target	1985:Q1	Model (Initial)
Consumption Basket (per cent of consumption)		
Non-tradable consumption	50.6	51.0
Home tradable consumption	35.4	32.8
Imported consumption	14.0	16.2
Labour Force (per cent of labour force)		
Employment	91.5	94.2
Unemployment	8.5	5.8
Employment (per cent of employed workers)		
Non-tradable employment	59.5	59.7
Home tradable employment	39.1	39.0
Commodities employment	1.4	1.3
Unemployment (per cent of unemployed workers)		
Non-tradable unemployment	54.1	51.9
Home tradable unemployment	44.1	46.8
Commodities unemployment	1.7	1.3

F Bayesian Estimates

In this Appendix we report the estimates for the stochastic component of the shocks. The prior on habit formation coefficient, h, is set as a beta distribution with mean of 0.71 and standard deviation of 0.16. We set a normal prior with a mean of 3 and a standard deviation of 0.5 for the investment adjustment cost, Υ''. Our choices of priors on the structural shock parameters follow the literature. The parameter that determines the persistence of shocks is drawn from a Beta distribution with mean 0.5 and standard deviation 0.2, while the standard deviation of the shocks is drawn from an Inverse Gamma distribution.

Table 1: Prior and Posterior Distributions for Shock Processes

	Prior distribution			Posterior distribution			
Parameter	Distribution	Mean	S.d.	Mean	Mode	5%	95%
Consumption habit and vancancy adjustment costs							
h	Beta	0.71	0.16	0.813	0.818	0.764	0.848
Υ''	Normal	3	0.5	3.431	3.461	3.318	3.521
Standard Deviations							
σ_ζ	Inv. Gamma	0.10	2	0.045	0.047	0.038	0.053
σ_{ξ_N}	Inv. Gamma	0.01	2	0.097	0.095	0.079	0.123
σ_v	Inv. Gamma	0.10	2	0.095	0.099	0.080	0.112
σ_z	Inv. Gamma	0.10	2	0.010	0.011	0.008	0.013
σ_{zH}	Inv. Gamma	0.10	2	0.042	0.043	0.037	0.047
σ_{zN}	Inv. Gamma	0.10	2	0.021	0.021	0.018	0.023
σ_{r^*}	Inv. Gamma	0.01	2	0.003	0.003	0.002	0.003
σ_{ψ_b}	Inv. Gamma	0.01	2	0.003	0.003	0.002	0.003
σ_χ	Inv. Gamma	0.10	2	0.064	0.062	0.051	0.081
AR Coefficients							
ρ_ζ	Beta	0.5	0.2	0.67	0.70	0.54	0.78
ρ_{ξ_N}	Beta	0.5	0.2	0.95	0.97	0.89	0.98
ρ_v	Beta	0.5	0.2	0.57	0.58	0.44	0.69
ρ_z	Beta	0.5	0.2	0.54	0.55	0.36	0.70
ρ_{zH}	Beta	0.5	0.2	0.88	0.90	0.81	0.95
ρ_{zN}	Beta	0.5	0.2	0.96	0.96	0.93	0.99
ρ_{r^*}	Beta	0.5	0.2	0.74	0.74	0.65	0.82
ρ_{ψ_b}	Beta	0.5	0.2	0.76	0.76	0.68	0.82
ρ_χ	Beta	0.5	0.2	0.89	0.89	0.81	0.96

G The Kalman Filter Equations

Take the state equation

$$y_t = C_t + Q_t y_{t-1} + G_t \varepsilon_t \tag{76}$$

and the observation equation

$$z_t = H y_t + v_t$$

Define $\mathbb{E}(\varepsilon_t \varepsilon_t') = \Omega$, $\mathbb{E}(v_t v_t') = V$ and

$$\begin{aligned} \hat{z}_{t|t-j} &= \mathbb{E}(z_t | z_{t-j}, \cdots, z_1) \\ \hat{y}_{t|t-j} &= \mathbb{E}(y_t | z_{t-j}, \cdots, z_1) \\ \Sigma_{t|t-j} &= \mathbb{E}(y_t - \hat{y}_{t|t-j})(y_t - \hat{y}_{t|t-j})' \end{aligned}$$

The recursion begins from $\hat{y}_{1|0}$ where the unconditional mean of y_1 is

$$\mathbb{E}(y_1) = \mu_1$$

where μ_1 is the steady state under the initial structure, that is $\mu_1 = (I - Q_1)^{-1} C_1$ and

$$\Sigma_{1|0} = \mathbb{E}(y_1 - \mu_1)(y_1 - \mu_1)'$$

implies $vec(\Sigma_{1|0}) = (I - Q_1 \otimes Q_1) vec(G_1 \Omega G_1')$. Presuming that $\hat{y}_{t|t-1}$ and $\Sigma_{t|t-1}$ are in hand then

$$\hat{z}_{t|t-1} = H \hat{y}_{t|t-1}$$

and the forecast error will be

$$u_t = z_t - \hat{z}_{t|t-1} = H(y_t - \hat{y}_{t|t-1}) + v_t$$

The latter implies that

$$\mathbb{E}(u_t u_t') = H \Sigma_{t|t-1} H' + V$$

Next, update the inference on the value of y_t with data up to t:

$$\begin{aligned} \hat{y}_{t|t} &= \hat{y}_{t|t-1} + \left[\mathbb{E}(y_t - \hat{y}_{t|t-1})(z_t - \hat{z}_{t|t-1})' \right] \left[\mathbb{E}(z_t - \hat{z}_{t|t-1})(z_t - \hat{z}_{t|t-1})' \right]^{-1} u_t \\ &= \hat{y}_{t|t-1} + \Sigma_{t|t-1} H' \left(H \Sigma_{t|t-1} H' + V \right)^{-1} u_t \end{aligned}$$

after using $\mathbb{E}\left(v_t \left(y_t - \hat{y}_{t|t-1} \right)' \right) = 0$. Equation (76) then implies

$$\hat{y}_{t+1|t} = C_{t+1} + Q_{t+1} \hat{y}_{t|t-1} + K_t u_t$$

where $K_t = Q_{t+1}\Sigma_{t|t-1}H'\left(H\Sigma_{t|t-1}H' + V\right)^{-1}$ is the Kalman gain matrix. This last expression, combines with Equation (76), implies that

$$
\begin{aligned}
y_{t+1} - \hat{y}_{t+1|t} &= C_{t+1} + Q_{t+1}y_t + G_{t+1}\varepsilon_{t+1} \\
&- \left(C_{t+1} + Q_{t+1}\hat{y}_{t|t-1} + Q_{t+1}\Sigma_{t|t-1}H'\left(H\Sigma_{t|t-1}H' + V\right)^{-1}u_t\right) \\
&= Q_{t+1}(y_t - \hat{y}_{t|t-1}) + G_{t+1}\varepsilon_{t+1} - Q_{t+1}\Sigma_{t|t-1}H'\left(H\Sigma_{t|t-1}H' + V\right)^{-1}u_t
\end{aligned}
$$

The associated recursions for the Mean Squared Error (MSE) matrices are given by,

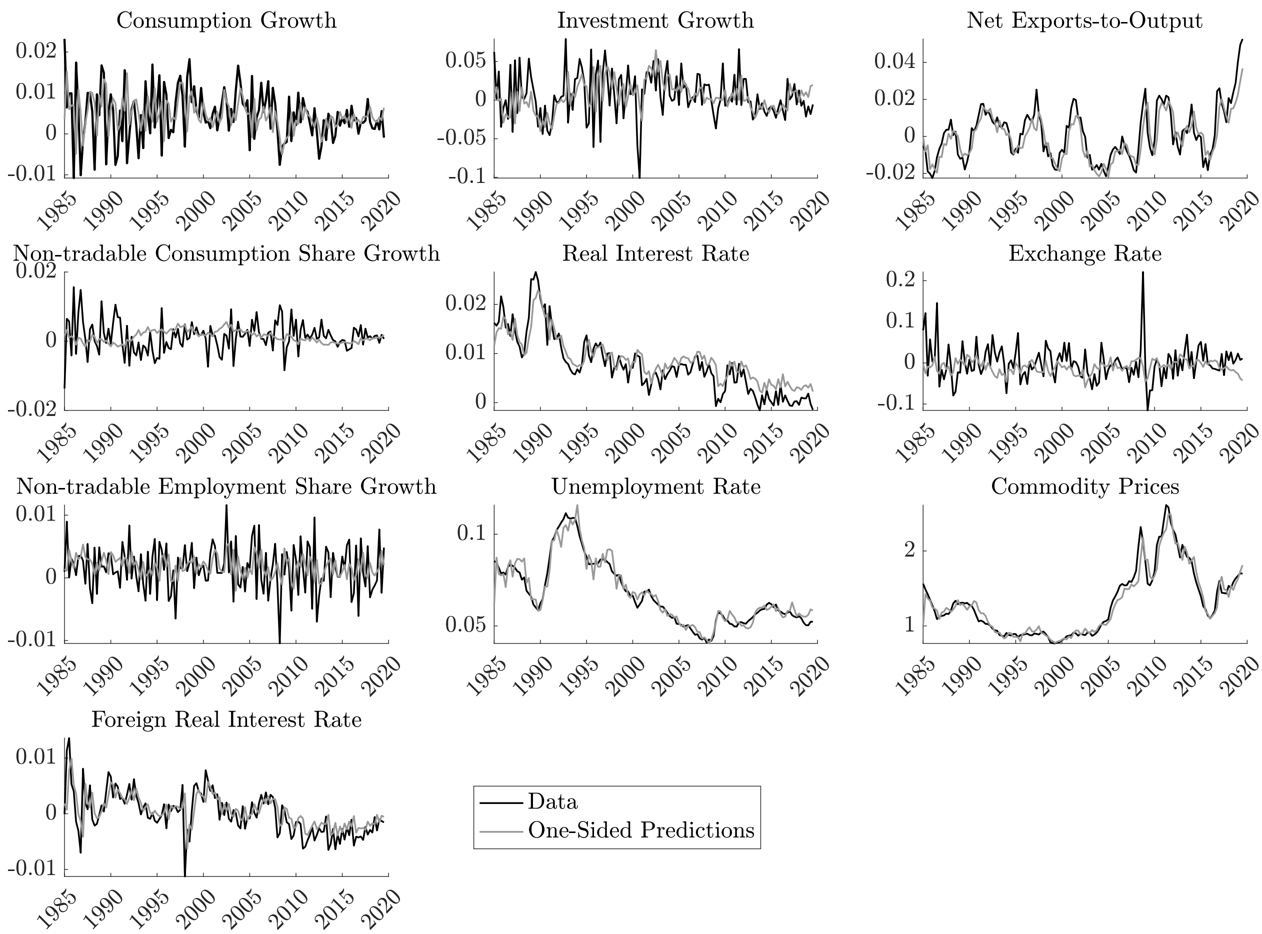
$$\Sigma_{t+1|t} = G_{t+1}\Omega G'_{t+1} + Q_{t+1}\left(\Sigma_{t|t-1} - \Sigma_{t|t-1}H'\left(H\Sigma_{t|t-1}H' + V\right)^{-1}H\Sigma_{t|t-1}\right)Q'_{t+1}.$$

If the initial state and the innovations are Gaussian, the conditional distribution of z_t is normal with mean $H\hat{y}_{t|t-1}$ and conditional variance $H\Sigma_{t|t-1}H' + V$. The forecast errors, u_t, can then be used to construct the log likelihood function for the sample $\{z_t\}_{t=1}^{T}$ as follows:

$$\mathcal{L} = -\left(\frac{n_z T}{2}\right)\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\det\left(H\Sigma_{t|t-1}H' + V\right) - \frac{1}{2}\sum_{t=1}^{T}u'_t\left(H\Sigma_{t|t-1}H' + V\right)^{-1}u_t$$

H Additional Results

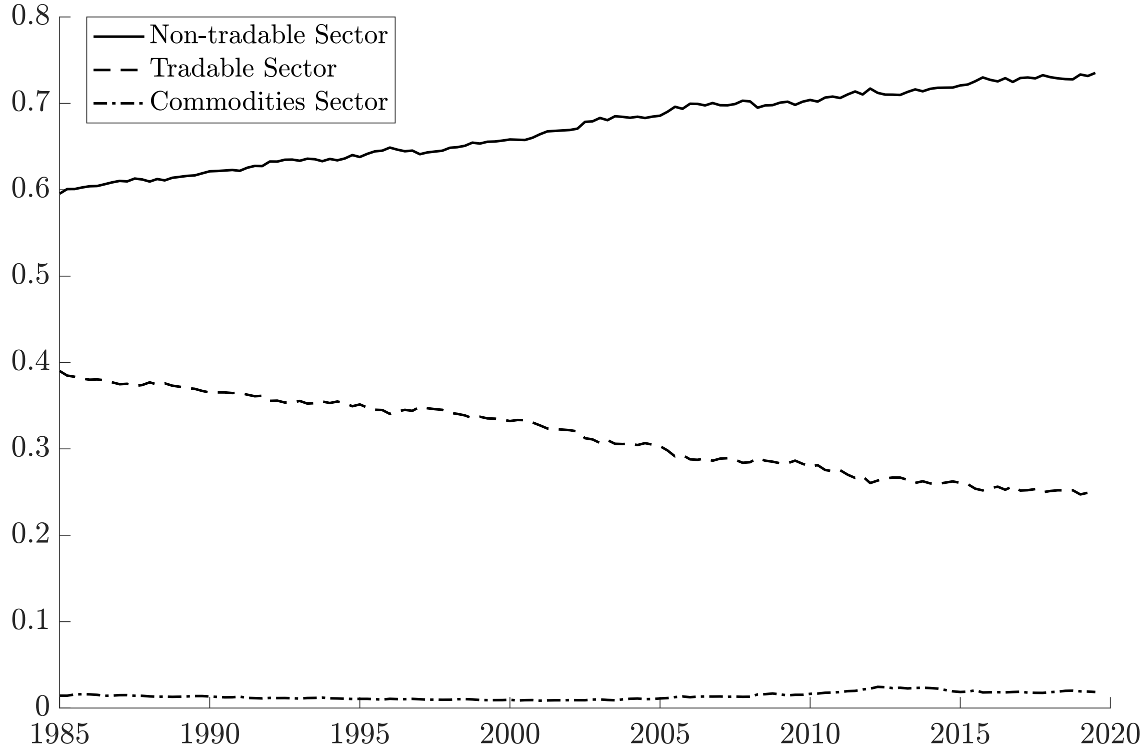
Figure 2: Data and One Sided Predictions



Sources: ABS, Authors' calculations; FRED; RBA.

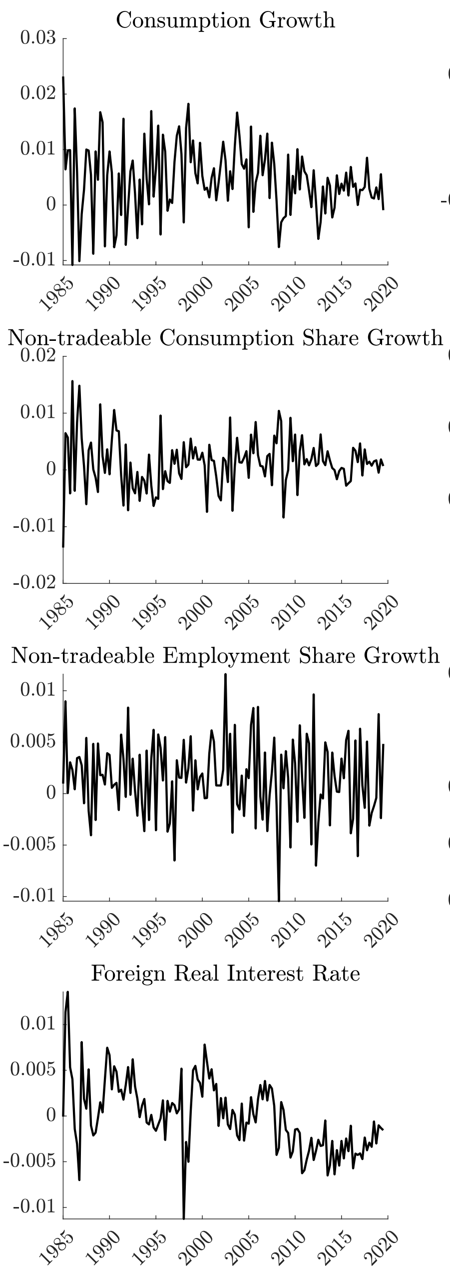
I Additional Figures

Figure 3: Employment Shares by Sector



Source: Authors' calculations; ABS.

Figure 4: Observed Data Used in Estimation



Source: Authors' calculations; ABS; FRED; RBA.

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